Numerous relative clauses

permutation invariance, anti-restrictiveness, triviality*

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Abstract

Two classes of collective predicates (*numerous*-like predicates and *gather*-like predicates) are known to give rise to different acceptability patterns in examples involving plural quantifiers. In this paper, I study different acceptability patterns that the two classes present in a novel domain, that of restrictive relative clauses. I argue that *numerous*-like predicates yield infelicity in restrictive relative clauses because of a certain algebraic property which, in combination with a pluralized predicate, results in triviality whenever maximality (such as the one arising from the definite article) applies on top. In the second part of the analysis, I show that systematic exceptions in which restrictive *numerous*-like predicates are not deviant, are borne out as a prediction of the present analysis, since they involve modification of unpluralized collective predicates. The proposed analysis will bear possible interesting consequences for the pluralization of predicates, the logicality of language and their interaction.

1 Introduction

The present paper aims at providing an analysis to two linked puzzles involving restrictive uses of *gather*(-like) and *numerous*(-like) predicates. The

*The main examples and the problems they constitute, together with important suggestions toward a solution, were proposed by Martin Hackl in a problem set for the course 'Introduction to Semantics'. I thank him, together with Omri Doron, Danny Fox, Patrick Elliott, Dean McHugh, Keely New, Viola Schmitt, Jad Wehbe, as well as all the participants to 'Introduction to Semantics', 'Workshop in Linguistics' and the 33rd European Summer School in Logic, Language and Information and six anonymous reviewers for precious suggestions and crucial feedback on an early version of this work.

first puzzle arises from the contrast shown by the following sentences:

(1) a. Jack only talked to the students that gathered.
b. #Jack only talked to the students that were numerous.

Whereas (1a) is a felicitous sentence in which the predicate *students* has been modified by *gathered* by means of a relative clause, (1b) - the result of substituting the predicate *gather* with *be numerous* - is somehow deviant.[1] These examples include *only* and *that* to make sure that the relative clause is interpreted as a restrictive relative clause. That is because in English, *that* can only introduce restrictive relative clauses, in contrast with *who*, which is compatible with both restrictive and non-restrictive ('appositive') readings (Schlenker, 2021; Stowell, 2005, a.o.). The appositive reading would be something akin to the following:

(2) a. Jack only talked to the students, who - by the way - gathered.
b. Jack only talked to the students, who - by the way - were numerous.

A preliminary question we have to ask is therefore whether the general infelicity of restrictive relative clauses involving the predicate *numerous* is specific to restrictive relative clauses or is rather linked to restrictiveness *per se*, regardless of how it is achieved. With respect to this issue, the following data from Italian is particularly helpful:

(3) a. Ho parlato con i numerosi studenti.
have.1SG talked with the.PL numerous.PL student.PL
I have talked with the numerous students.
b. ??Ho parlato con gli studenti numerosi.
have.1SG talked with the.PL student.PL numerous.PL
I have talked with the numerous students.

The minimal pair above exploits a typical Romance phenomenon: modifiers in pre-nominal positions are linked to non-restrictive interpretations, while post-nominal modifiers are ambiguous between the two readings (Cinque,

[1] There are some speakers that upon reflection manage to find (1b) acceptable 'imagining a scenario in which there are different groups of students'. I claim that in this case *students* is contextually interpreted as *gathered students*, like an implicit version of the later example (5). I don't discuss here whether this interpretation can be due to a cover analysis or to an implicit *gathered*. However, I believe that for the intended purposes these two possible analyses are equivalent, even though the cover analysis might require additional assumptions on the application of relative clauses.

2003; Morzycki, 2008).[2] The contrast between (3a) and (3b) signals then that the infelicity of the predicate *essere numeroso / to be numerous* is really tied to restrictive interpretations in general, rather than limited to (restrictive) relative clauses.[3] The English counterpart of (3a) is also good and indeed conveys a non-restrictive reading of *numerous*.

(4) I talked to the numerous students.

Given the data, there must be something that blocks *numerous* from being restrictive, namely from selecting a specific subset of a given set.[4] The contrast observed in (1) becomes even more compelling when compared with the second puzzle I will discuss, which arises from the following sentence:

(5) Jack only talked to the gathered students that were numerous.

This pivotal data seems to contradict the generalization I stated above: (5) shows that *numerous*-restrictive relative clauses can be felicitous; in particular, they are perfectly fine if the predicate they combine with is a complex predicate of a certain kind, in our case if *student* has already been modified by some other predicate, *gather*. I will show that what makes these complex predicates special is a specific internal structure, required for the denotation of these predicates to felicitously combine with *numerous*. Crucially, I will derive the felicity of (5) as a prediction of my analysis for the infelicity of (1b), once it is assumed that pluralization is a rather free operation that can be applied to an *et*-type node different from the external merging site of the predicate.

To summarize the goal of this paper, we can take away two main observations and two questions, respectively, from the examples presented so far:

1. **Observation 1**: the modifier *numerous* is generally fine as non-restrictive but not as restrictive;
   Question 1: what is preventing it from being restrictive?

2. **Observation 2**: there are exceptions to Observation 1.
   Question 2: what makes these exceptions 'special'?

[2] Crucially, I do not commit here to any specific syntactic or semantic analysis of the contrast between the two interpretations, beyond the idea that restrictive interpretations select a subset of the predicate they combine with.

[3] Unless post-nominal modifiers are analysed as some 'compressed' relative clauses.

[4] We don't delve here into the thorny question of the proper semantics for restrictiveness and non-restrictiveness, and we just treat restrictive modification as the intersection of the sets denoted by the two involved predicates.

To answer the first question, I claim that restrictive readings are blocked whenever a *permutation invariant* predicate like *numerous* applies to complete join semilattices (i.e. 'starred' - namely, pluralized - predicates). This would lead to triviality: the individual resulting from the application of the relative clause has no potential to differ (be restricted) from the individual that would have been denoted without the relative clause.[5]

To answer the second question, I will build on the intuition that (5) involves several groups of gathered students. I will argue that while most complex predicates (like *smiling students*) preserve a complete structure that displays all possible sums of the individuals in the domain, others (like *gathered students*) have the possibility to give rise to other, 'incomplete structures'. In particular, I will claim that *gathered* changes the complete join semilattice structure of *students* when *gathered* is not itself pluralized. I will assume that pluralization comes from a syntactic operator, the star operator (Link, 1987), in line with Sternefeld (1998) and Beck and Sauerland (2000). On the one hand, I assume that pluralization can apply to every base-generated and complex predicate; on the other hand, I assume pluralization to be optional, or, more precisely, omitted if its application leads to triviality. Let us now start from an introduction of the relevant properties of *gather*-like and *numerous*-like predicates that have been discussed in the existing literature. I will claim that while these properties do not constitute the solutions to our puzzle - as well as to those already discussed in the literature - they contain important insights towards the analysis that needs to be provided.

2 *Gather*, *be numerous*, and their relevant properties

Both *gather* and *be numerous* can be classified as collective predicates, for they only accept plural individuals in their domain. However, it has been known for a while now that a more fine-grained distinction is needed within the class of collective predicates, since *gather* and *numerous* give rise to different acceptability patterns when combined with plural quantifiers (Champollion, 2010; Dowty et al., 1987; Kroch, 1974). These patterns resemble

[5] As I will show, the solution is marginally linked to the *anti-monotonicity constraint* proposed in Amiraz (2021). However, I do believe that the interplay between the structure of plurals and *permutation invariant* predicates is the real notion at play here. I will discuss the connections between my analysis and the 'anti-monotonicity' proposal of Amiraz (2021) in the Appendix.

the contrast presented in (1), as only so-called *gather*-type predicates can felicitously combine with plural quantifiers, as displayed in (6) and (7):

(6) a. All the students gathered.
b. #All the students are numerous.

(7) a. Most of the students gathered.
b. #Most of the students are numerous.

A couple of intuitions on the properties that are linked to this problem are generally shared by the literature, and, following Kuhn (2020), they can be summarized as follows:

1. *numerous*-like predicates denote properties of groups as such;

2. *gather*-like predicates license 'distributive sub-entailments'.

Property 1 states that *numerous*-like predicates do not hold (or are not guaranteed to hold) for subparts of what they are predicated of. Consider for example the case in which a group of 5 students can be described as numerous. There is no guarantee that a group consisting of only 4 of those 5 students will also be considered numerous. In other words, if a certain sum is in the denotation of a *numerous*-like predicate, there is no guarantee that a proper part of that sum will also be in the denotation of that predicate. The opposite of this property is the licensing of 'distributive sub-entailments' which applies to *gather*-like predicates as stated in 2. In Kuhn's (2020) terms, the distinction between the two classes of predicates reflects the properties of a mass/count distinction. In particular, in the same way every part of something that is water is water itself, *gather*-like predicates have the crucial property of holding for any plural subpart of the individual they are predicated of.

Definition 1 *[Distributive sub-entailments] If a predicate holds for a plurality x, then it holds for any y s.t. $y \leq x$ and $\neg Atom(y)$.*[6]

While these two core properties have been proven to be very useful in discussing the *gather*/*numerous* distinction, we also know that they are not enough to capture the heart of the distinction, since - as they are stated - they suffer from both overgeneration (see Kuhn (2020)) and undergeneration (see Amiraz (2021) and Kuhn (2020)). In particular, on the one hand, we

[6] First proposed in Dowty et al. (1987).

have *numerous*-like predicates that would be incorrectly classified as *gather*-like predicates, since they license distributive sub-entailments as defined in Def.1. This is for example the case of *few in number* (Kuhn, 2020):

(8) a. The students were few in number.
b. #All of the students were few in number.
c. #I only talked to the students that were few in number.

Note in fact that if *few in number* holds for a plurality x, it will also hold for any non-atomic part of x. Nonetheless, *few in number* does generate the *numerous*-like oddities in both the domain of plural quantifiers and restrictive relative clauses. On the other hand, there are predicates that do not necessarily license sub-entailments for every sub-part but they still pattern with *gather*, like *hold hands* (Kuhn, 2020):

(9) a. The students held hands.
b. All of the students held hands.
c. I only talked to the students that held hands.

Crucially, if we imagine ten students standing in a circle in which each student holds hands with the students on her left and on her right, *hold hands* would hold for the sum including all ten students, but not necessarily - or rather 'necessarily not' - for every sub-part constituted by three students out of those ten. For these reasons, Kuhn (2020) proposes *stratified reference*, the most recent and refined variant of Def.1, which aims at capturing properties 1 and 2 under an event semantics approach.

While Amiraz (2021) shows that Kuhn's (2020) analysis might still suffer undergeneration, it has to be recognized that the intuitions behind stratified reference have proven to be very useful - at least in the literature. It would be nice if such intuitions could be related to certain fundamental aspects of the grammar that would already predict for independent reasons *numerous*-like predicates to result in infelicity when used in certain environments. The crucial question therefore is: can we re-think the two literature intuitions about the *gather*/*numerous* distinction in terms of sum formation to detect the deep link between the two kinds of predicates and pluralities as such?

2.1 Permutation invariance and anti-restrictiveness

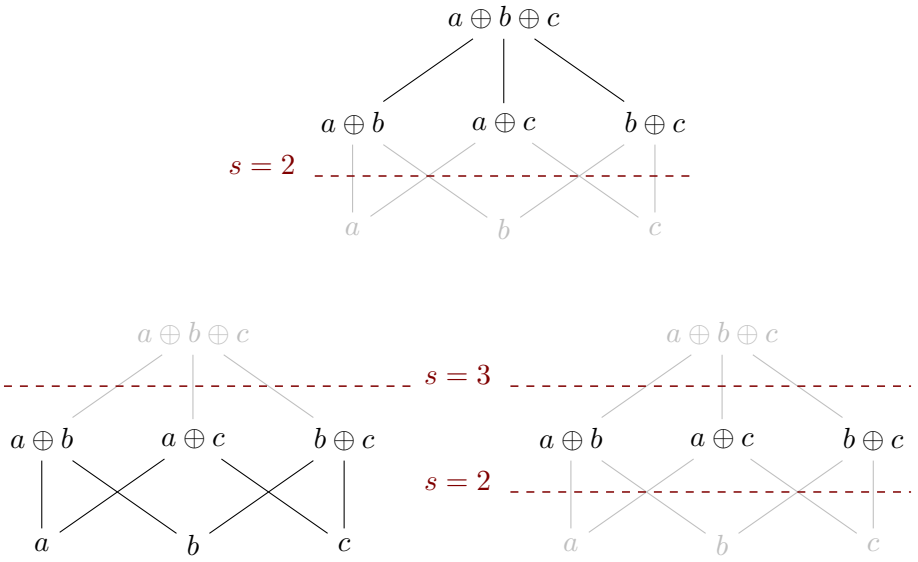
While in this paper I won't engage directly with the plural quantifier puzzle, I use the case of relative clauses as an opportunity to re-think the two basic intuitions that its literature has focused on. In the conclusions, section §5,

however, I will briefly sketch how the ideas proposed in what follows can be applied to the plural quantifier puzzle. Now, as I said in section §2, the literature teaches us that a fundamental property of *numerous*-like predicates is that they 'denote properties of groups as such'. This observation goes in the right direction, but while the literature has focused on interpreting 'denoting properties of groups as such' as 'being unable to guarantee the licensing of sub-entailments', I believe that the pivotal notion at play is that these predicates are 'blind to differences'. They denote properties of 'groups as such' in the sense that a certain sum is in the denotation of the predicate regardless of any specific attribute of the atoms that compose the sum. In other words, these predicates do not 'care' about whether a certain atom instead of another is in the plurality they are predicated of. Take for example the case of *numerous*: it only checks whether a given plurality has a certain cardinality, regardless of the properties of the specific members that constitute such plurality. To say that $a \oplus b \oplus c$ are a *numerous* plurality, we are not concerned with any of the properties of a, b and c, it suffices to say that the plurality $a \oplus b \oplus c$ exists (and passes the 'numerosity' threshold that is contextually established), i.e. we are only concerned with the fact that we can group these atoms together with respect to some property they share. For instance, if we substituted c with d, the numerosity of the plural individual would not change, and $a \oplus b \oplus d$ would again be in the denotation of *numerous*. In contrast, when *gather* is predicated of $a \oplus b$, it is crucial that a and b participated in a gathering event together (hence the event semantics used by Kuhn (2020)), and substituting b for another element f, will not necessarily guarantee that $a \oplus f$ is in the denotation of *gather* too. We can call this property *permutation invariance*.

Definition 2 *[Permutation invariance] A predicate P is permutation invariant if, for all individuals $a_1 \oplus ... \oplus a_n$ and $b_1 \oplus ... \oplus b_n$ such that $|a_1 \oplus ... \oplus a_n| = |b_1 \oplus ... \oplus b_n|$, we have $P(a_1 \oplus ... \oplus a_n) \Leftrightarrow P(b_1 \oplus ... \oplus b_n)$.*

A predicate is permutation invariant if, in case it holds of an individual with n number of elements, it also holds for any other individual in its domain with n number of elements. Following the discussion above, it is interesting to think about it as the property that allows us to substitute an atom in a sum for another atom and be guaranteed that the value of the predicate applied to the sum will not change. Reading the definition, it clearly emerges how this is exactly the defining property of those predicates that denote cardinalities, such as *be numerous/few in number/more than 5/exactly 4/less than 3/etc.* These are the predicates involved in our

puzzle and they bear an intrinsic relation to the sum formation operator, since they check whether a certain threshold has been reached with respect to sum formation.[7] To help us visualize this we could say that permutation invariant predicates are those that establish a horizontal cut-off when they modify other predicates. Consider for example the case of a pluralized predicate (visualized with a graph representing their traditional semi-lattice structure (Link, 1987)), modified with - respectively - *numerous/more than one*, *few/less than three*, *exactly two*:



Note that this property equally applies to predicates that are monotonic with respect to sum formation in either way (*numerous* vs *few in number*), and not monotonic at all (*exactly two*). It follows that neither super-entailments nor sub-entailments are the crucial notions at play here. Now, consider that, given a predicate P, its pluralized version is built by forming all possible sums of the elements in the denotation of the predicate. This is obtained via the star operator *, where $^*P$ is the smallest function s.t.

$$\forall x[P(x) = 1 \rightarrow^* P(x) = 1] \text{ and}$$
$$\forall x, y[^*P(x) = {}^*P(y) = 1 \rightarrow^* P(x \oplus y) = 1],$$[8]

often interpreted as the contribution of the plural morpheme *-s*. As said

[7] As said above, I will discuss in the Appendix how this idea is indebted to Amiraz (2021).

[8] Martin Hackl, lecture notes.

a few lines above, given a set of entities, * returns the set containing all possible sums of those entities. This implies that every equivalence class based on cardinality (i.e. every 'horizontal layer' of the semilattice) is a subset P' of the predicate $^*P$ that contains at least one element per atom. As the pictures in the previous page show, it follows that whenever a permutation invariant predicate modifies a pluralized predicate, the denotation of the resulting complex predicate will contain, for any atom x, at least one individual X containing x as a part, since permutation invariant predicates establish 'horizontal cut-offs'. I call this - the property that describes the result of combining a permutation invariant predicate with a pluralized predicate - *anti-restrictiveness*.

Definition 3 *[Anti-restrictiveness] A predicate P is anti-restrictive with respect to another predicate Q iff (i) for all x s.t. $Q(x)$ and $Atom(x)$, there exists y such that $x \preceq y$[9] and $P(y)$ or (ii) there is no y such that $P(y)$, for every model.*

In other words, a predicate P is anti-restrictive with respect to another predicate Q if and only if, no matter how we pick the model, either every atom in Q is part of some sum in P or P is empty. Talking loosely, either there is no numerous group of students or every student is part of at least one numerous group of students. In the next section, I will provide an analysis for how these properties play a crucial role in blocking restrictive interpretations of *numerous*-like predicates whenever maximality, such as the one originating from the definite article, applies on top.

3 The analysis, part I: unrestricted triviality

Consider again the initial example (1b), repeated here as (10):

(10) # Jack only talked to the students that were numerous.

In this section, I aim at showing how, because of the algebraic properties of plurals, whenever *numerous* tries to restrict a pluralized predicate, it inevitably ends up receiving a non-restrictive interpretation, which results in ungrammaticality whenever the syntactic environment is only compatible with restrictive readings.

[9] Where $x \preceq y$ means 'x is part of y'.

3.1 Permutation invariance, anti-restrictiveness, triviality

As I did in the previous paragraph, I will assume that the denotation for *students* is the complete join semilattice due to Link (1987). I also assume that this is obtained interpreting the plural morpheme *-s* as the star operator, which given a set of entities returns the set of all possible sums of the elements of that set. These ingredients are used here together with the following definition for *numerous*:

$$[\![numerous]\!]^{g_c} =$$
$$\lambda x : x \in dom(|\ \ |).|x| \geq \text{the contextually supplied numerosity standard}$$

In simple terms, *numerous* denotes all those plural individuals whose cardinality is above a certain threshold. A visual representation of *students that are numerous* is then shown in Fig.1, where the numerosity threshold has been set to 2 as in the previous section, for the sake of simplicity.

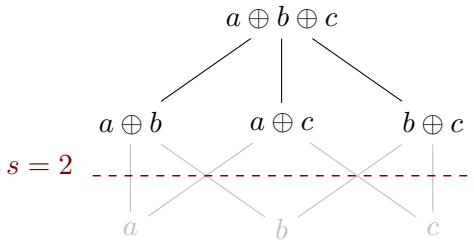


Figure 1: $[\![students\ that\ are\ numerous]\!]^{g_c} = \{a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c\}$

With no additional contextual restriction on how we can pick plural individuals, *any* group with cardinality 2 or more is going to be in the denotation of *numerous* [PERMUTATION INVARIANCE].[10] This means that for any x such that x is a (student) atom, there is at least one y such that y is a numerous individual and $y \succeq x$ [ANTI-RESTRICTIVENESS]. As said in the previous section, anti-restrictiveness arises whenever a pluralized predicate is modified by a permutation invariant predicate. Note that pluralization on *student* needs to take place, as *numerous*-like predicates are collective predicates, i.e. they are only defined for plural individuals so they can only modify predicates that have plural individuals in their denotation. Now, the

[10] In different terms, if one wanted to claim that $a \oplus b$ from the previous figure is a numerous individual, he wouldn't be able to hold that $a \oplus b$ is a numerous individual without admitting that $a \oplus c$, $b \oplus c$, and $a \oplus b \oplus c$ are numerous individuals too.

last element of the analysis is the definite article which I assume to denote the following meaning, adapted from Sharvy (1980):

$$[\![the]\!]^{g_c} =$$
$$\lambda P : P_{\langle e,t\rangle} \& \exists x[P(x) \& \forall y[P(y) \rightarrow y \preceq x]].\iota z[P(z) \& \forall y[P(y) \rightarrow y \preceq z]]$$

The definite article picks the maximal individual satisfying a certain property, once the presupposition that such individual exists is satisfied. For instance, when applied to the denotation I presented above, *the* would pick $a \oplus b \oplus c$:

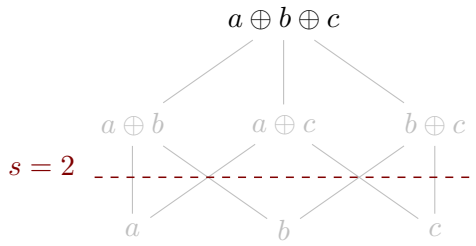


Figure 2: $[\![$*the students that are numerous*$]\!]^{g_c} = a \oplus b \oplus c$

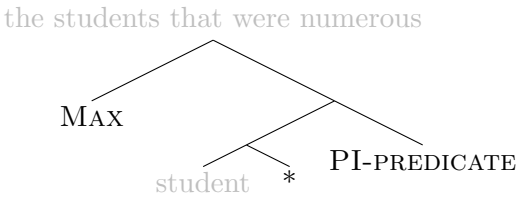
Crucially, $a \oplus b \oplus c$, the maximal element in the denotation of *students that are numerous* is the same maximal element in the denotation of *students*. This means that *the students that are numerous* has the same denotation as *the students*. In other words, the denotation of the mother node to the definite article and the NP is the same with or without the restrictive relative clause. The restrictive construction loses its purpose, as it does not restrict anything. Restrictive constructions of the form *the As that are B* might be contingently trivial if in a certain model it so happens that all elements of A are in B. However, in the case of permutation invariant predicates, relative clauses are necessarily, and thus pathologically trivial.

Definition 4 *[Trivial restriction] The application of a restrictive construction is trivial if the denotation of the maximal projection of the noun modified by the restrictive construction (the DP) is unaffected by the modification in all models.*

The idea behind this principle is that a sentence is infelicitous if a structure contains an element that is pathologically, i.e. irrecoverably, meaningless for the structure. If a restrictive relative clause cannot restrict the

noun it modifies no matter the situation, the sentence containing the relative clause is marked as deviant.

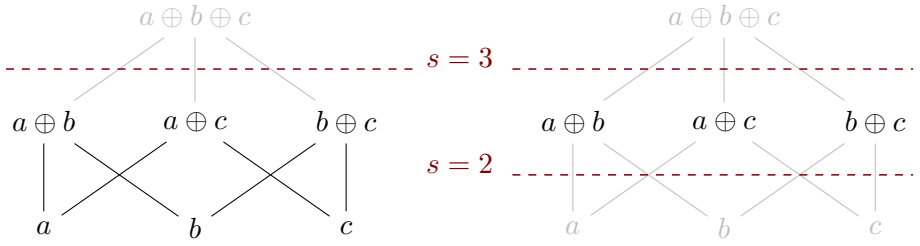
At this stage, I don't commit to any specific theory on how exactly triviality leads to unacceptability. Let me just briefly mention two possibilities. On the one hand, we could imagine that restrictive relative clauses bear a presupposition that requires them to effectively restrict the DP that results from their modification. In this sense then, permutation invariant predicates modifying pluralized predicates would result in systematic presupposition violations. The underlying assumption is that this RESTRICT! presupposition would need to be checked somehow at the DP node as well. This idea was suggested to me by Martin Hackl (p.c.). On the other hand, a possible alternative is based on the line of work on L-triviality in the spirit of Del Pinal (2019) and Gajewski (2002), who assume that ungrammaticality arises when triviality affects 'logical skeletons', namely LFs modulo non-logical terms. Under this view, we should conjecture that the definite article's maximality and the cardinality (permutation invariant, PI) component of *numerous*-like predicates are logical terms visible to the deductive system, as well as the star operator:



Sentences would then become infelicitous whenever maximality applies on top of a permutation invariant predicate that modifies a node that has * as one of its daughters. It has however to be stressed that at the current state of the project these possible solutions are mere stipulations. Moreover, there seems to be the need for a mechanism that explains why non-restrictive clauses necessitate a dedicated syntax and cannot be found in syntactic environments associated with restrictive interpretations more generally (Danny Fox, p.c.; see also Schlenker 2021). While the L-triviality route might be an interesting line of research to shed light on the actual mechanisms that connect certain trivialities to unacceptability judgments, definitive evidence that we are indeed facing a case of L-triviality is yet to be established. Before moving now to the analysis of (5) as a typical example of a case in which restriction by *numerous*-like predicates does not end up being unacceptable, a comment on the definite article and other *numerous*-like predicates has to be made.

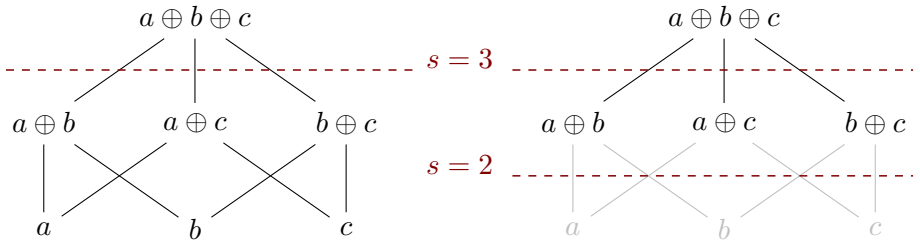
3.2 A note on the definite article

The fact that the definite article *the* could apply to *students that are numerous* has been taken for granted in the previous section. In fact, since *numerous* is monotonic with respect to the parthood relation, the maximal sum of the predicate it modifies will always be in its denotation (unless it is completely empty) and this ensures that the presupposition of the definite article can be met. However, we have seen before that there are permutation invariant predicates, like *few in number* and *exactly two* that 'cut off' the maximal sum from their denotation.



These predicates however still suffer from the triviality problem described in the previous paragraph, if they want to provide a suitable denotation for the definite article to apply. Consider in fact that if a predicate does not contain any maximal element, then the definite article fails to apply, since its presupposition is not met and it cannot pick any individual at all. The complex predicate resulting from the application of the *few in number/exactly 2*-relative clause to the original predicate of the NP (*students*, in our example) has then to be pluralized (i.e. closed under sum formation via *), to satisfy the existence of a unique maximal element and ensure composition with the definite article. However, if any atom is part of at least one individual in the denotation of *students that are few in number/exactly 2* [ANTI-RESTRICTIVENESS], then the maximal element resulting from the application of star to *students that are few in number/exactly 2* will just be the maximal element of *students*, i.e. the sum of every atomic individual in the predicate's set.

The definite article would then pick that very same element ($a \oplus b \oplus c$, in our example) that it would have picked even if the relative clause was not there, causing again a triviality-derived infelicity. Let us now move to the analysis of those exceptions in which restrictive modification by permutation invariant predicates goes through. As we will see, in those cases, the fact that pluralization can happen after modification to ensure composition with the definite article is going to be the key feature that rescues the acceptability



of the sentence.

4 The analysis, part II: starring, most predicates but not *gather*

In section §1, I introduced (5), repeated here as (11), an example in which restrictive modification by *numerous* is not marked as deviant:

(11) Jack only talked to the gathered students that were numerous.

The scenario described by speakers when confronted with (11) is one in which there are multiple student groups, some of them being numerous, and some of them not. I want to build on this intuition to explain what is special about *gathered students*. The key idea is that modifying a pluralized predicate like *students* with a purely collective predicate like *gather* can give rise to a non-pluralized collective predicate, which can felicitously be modified by permutation invariant predicates like *numerous*.

4.1 Collective predicates and structural modifications

As stated above, *numerous*-like predicates will end up being trivial whenever they are modifying complete join semilattices. This means that they would result in infelicity whenever they are modifying simple pluralized predicates (*students)*, or intersections of pluralized predicates (like *smiling students*)[11], with maximality applying on top. As said in section §3, if these distributive predicates were not pluralized, modification by *numerous* would fail from the start because *numerous*, it being a plural predicate, cannot be applied to atomic entities.[12] Now, given what has been said so far, one could expect

[11] Provided that the intersection of two pluralized predicates is itself a pluralized predicate.

[12] As a side comment, this might be the reason why speakers object that 'singular students cannot be numerous' when confronted with our initial example (1b). Probably

numerous-like predicates to be able to felicitously modify predicates that can provide the right sort of individuals (plural entities), without generating a complete join semilattice. This is the case for collective predicates when they are not pluralized: they contain plural entities in their denotation by definition but not necessarily all of their sums. The predicate *gather* is one of such purely collective predicates. In the same way that a non-pluralized distributive predicate like *smile* would have in its denotation only the atomic individuals that smile, a predicate like *gather* would have in its denotation only the sums of people that gather together (and their sub-entailments). For instance, if the students gathered in one room and the professors in another, the sum of students and professors would not be in the denotation of unpluralized *gather*. In other words, modification with unpluralized purely collective predicates is not semilattice preserving. As a consequence, the triviality problem with restrictive *numerous* does not arise, since anti-restrictiveness does not apply to such cases. Let me provide a concrete scenario that shows why (11) is felicitous. I have already said that the scenario described by speakers when confronted with (11) is one in which there are multiple groups of students, some of them being numerous, and some of them not. Consider then a concrete instance of that scenario: a, b, c, e and f are students that gathered as two separate groups, a, b and c gathered together in room A, while e and f gathered together in room B. Assume '3' as the contextually supplied cardinality necessary for *numerous* to hold. If *numerous* modifies then *gathered students* (in Fig.3), where *gather* is unpluralized, the result yields $a \oplus b \oplus c$ as the only numerous individual in the set.

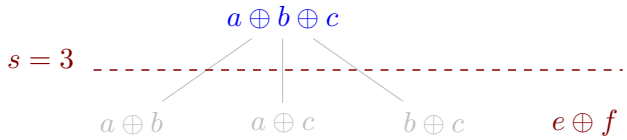


Figure 3: $[\![\textit{gathered students}]\!]^{g_c} = \{a \oplus b, a \oplus c, b \oplus c, e \oplus f, a \oplus b \oplus c\}$
$[\![\textit{gathered students that are numerous}]\!]^{g_c} = \{a \oplus b \oplus c\}$

In this structure, restriction by *numerous* manages to go through effectively, selecting a numerous individual that excludes $e \oplus f$: there is no numerous y such that $e \preceq y$ or $f \preceq y$. As a consequence, when we feed *gathered students that are numerous* to *the*, the definite article will pick $a \oplus b \oplus c$,

pluralization is removed to avoid triviality, but a mismatch between categories of entities (singular vs plural) arises.

and not $a \oplus b \oplus c \oplus e \oplus f$. This time the relative clause meaningfully and successfully restricts the predicate: the maximal individual selected at DP in the structure with the relative clause is strictly smaller than the maximal individual that would have been selected if the clause was not there. The assumption that permits this analysis is that pluralization can be freely omitted to avoid triviality (i.e. pluralization does not come with lexical entries but it is a subsequent operation). Now, consider the slightly more complicated case in which multiple groups of gathered students can be considered numerous.



Figure 4: $[\![\textit{gathered students that are numerous}]\!]^{g_c}$

It seems that, even in such scenario, (11) can be uttered felicitously to mean that Jack only talked to a, b, c, m, n and o, but not to e and f. Notice that there is no maximal element in the denotation displayed in Fig.4 and therefore if this denotation were to be fed as is to the definite article, the presupposition of the latter would not be met. This however is not a problem, assuming that pluralization can be applied *after* modification by *numerous* takes place, but before composition with the definite article. The LF of (11) would therefore be the following, where pluralization takes place on *students*, to compose with the collective predicate *gather*, and before the definite article, to meet its presupposition in case there are multiple groups of gathered students that were numerous.

(12) Jack only talked to [the [* [[[gathered] [* [students]]] [that were numerous]]]]

In this case, *the gathered students that are numerous* would then denote $a \oplus b \oplus c \oplus m \oplus n \oplus o$ and modification by *numerous* would have applied meaningfully again, since $e \oplus f$ have been discarded. In this case too, I do not commit to any specific machinery that governs the application of star, apart from recognizing that avoiding triviality plays a role in its omission at certain nodes. On the one hand, we could imagine pluralization to be a rather free operation that can be applied (or 'delayed') to any node in which a new predicate is formed (e.g. *gather*, *gathered students*, *gathered students that are numerous*, ...). On the other hand, more radically, we

could conjecture that pluralization is strictly linked to the plural morpheme and can only apply right after the predicate is merged, but the definite article always carries a star operator that applies before predicates are fed to the definite article. This star could act as a 'safety' measure, since it would have no effect for predicates that already have a maximal element (including singular nouns) and would only make a real contribution in those rare cases in which the fed predicate does not have a maximal element yet, as the ones we just proposed. If the second possibility is true however, we should never observe a presupposition failure for cases in which the definite article applies to collective predicates caused by having more than one possible entity to pick. In this respect, certain examples in Fintel et al. (2014) might constitute an interesting counterexample, suggesting that the first option is more sound. I leave this debate to future work. Before moving to the conclusions, I want to refine the notion of anti-restrictiveness, as the way it was defined in section §2, although having been very useful for explanatory purposes, is slightly imprecise.

4.2 A note on anti-restrictiveness

Note that the original definition of *anti-restrictiveness* provided in §2.1 makes reference to atoms. This means that, strictly speaking, that definition would not apply to *gathered students* even when this complex predicate is pluralized, that is because no atom is ever going to be in the denotation of *gathered students* in the first place, *gather* being a collective predicate that is only defined for plural individuals, as said in the previous section. The previous definition of *anti-restrictiveness* was provided for the sake of argument simplicity and clarity, but a more refined and maybe intuitive version can now be stated:

Definition 5 *[Anti-restrictiveness - refined] A predicate P is anti-restrictive with respect to another predicate Q iff (i) for all x s.t. there exists z such that $x \preceq z$ and $Q(z)$, there exists y such that $x \preceq y$ and $P(y)$ or (ii) there is no y such that $P(y)$, for every model.*

In simple words, Def.5 says that a predicate P is anti-restrictive with respect to another predicate Q if P is empty or if every atom appearing in some sum in the denotation of Q is also necessarily part of some sum in the denotation of P. Again, the idea behind it is that such predicates are unable to discard any atom that would end up being part of the maximal individual picked up by the definite article. Let us now move to the final section, in

which the conclusions of the paper will be drawn, together with directions for future research beyond those already expressed in previous sections.

5 Conclusions and Future Work

In this work I have provided an explanation to the contrast observed in (13a) and (13b):

(13) a. #Jack only talked to the students that were numerous.
b. Jack only talked to the gathered students that were numerous.

I claimed that the unacceptability of (13a) is due to the anti-restrictiveness of *numerous*-like predicates when applied to certain structures. Any atom of a predicate modified by a permutation invariant predicate cannot be excluded if the original predicate is pluralized, since every atom appears as part of some individual in every layer of the complete join semilattice generated by the predicate's pluralization. This implies that any proper restriction is forbidden whenever maximality applies on top, given that the selected maximal element from the *numerous*-modified predicate will be the sum of every single atom of the original predicate. I then explained how (13b) is a correct prediction of the presented theory, observing that unpluralized collective predicates can denote pluralities without being closed under sum formation. The underlying assumption of the system is that pluralization can be omitted and applied at a later node than the one in which the predicate is externally merged. The application of star in felicitous sentences, like (13b), is driven by the simultaneous satisfaction of two semantic and logical properties: meeting the presupposition of the definite article, and avoiding triviality. Such a reading is impossible for infelicitous sentences like (13a), since removing pluralization to avoid triviality leads to a mismatch between the atomic individuals of distributive predicates like *students* and the plural individuals to which *numerous*-like predicates can apply. For future research, I would like to fully explore the advantages of the present approach based on keeping predicates unpluralized over a possible alternative analysis based on covers, in the style of Schwarzschild (1996). While covers might be an appealing way to model the reading in which *gathered students* denotes multiple groups of students, it seems that such an approach would still require assumptions on pluralization similar to those made here. In particular, the cover approach could face some challenges in explaining how the relative clauses compose with denotations resulting from 'non-pluralized' covers, while still passing on a pluralized meaning to

the definite article. On these lines, in the near future I would like to extend the analysis to the other quantificational constructions that present a contrast between *numerous*-like and *gather*-like predicates: plural quantifiers and non-proportional partitive constructions (Amiraz, 2021; Champollion, 2010; Dowty et al., 1987; Kroch, 1974).

(14) # [[Most of [the students]] are numerous].

Interestingly enough, modifying *students* with *gather* does not change the judgment of the sentence:

(15) # [[Most of [the gathered students]] are numerous].

Although I don't possess yet a full account of these sentences, we can see why examples like (15) would be problematic for any account that aimed at explaining such examples, or the ones discussed in this paper, with the use of covers, including Kuhn (2020). That is because such accounts should explain why (15) cannot mean that most groups of gathered students are numerous. On the other hand, extending this paper's account to the plural quantifiers puzzle would also require at least one further distinction in the domain of *numerous*-like predicates, since certain predicates that pattern with *numerous* in the plural quantifiers puzzle, pattern with *gather* in the restrictive relative clauses examples:

(16) a. ??Most of the students formed a circle.[13]
b. I only talked to the students that formed a circle.

Such an account could then teach us that the literature on the plural quantifiers puzzle has grouped together two different types of predicates that might generate unacceptability judgments for two different reasons. Finally, another domain of research in which the ideas proposed here can be extended is that of generics, on which I am currently working with Janek Guerrini. It's been said above that triviality arises whenever maximality applies on top of an anti-restrictive construction. We should therefore expect unacceptability judgments to show up in other environments that display maximality. This seems to be the case of bare plural generics:

(17) Gathered students are loud.
a. *Can mean:*

[13]The deviancy of this sentence is debated, as Kuhn (2020) reports *form a circle* to be a *numerous*-like predicate, but other scholars disagree (Yoad Winter, p.c.).

In general, if a group of students is gathered, it is loud.

(18) Numerous students are loud.

a. *Cannot mean:*
In general, if a group of students is numerous, they are loud.

b. *Can mean:*
The number of students that are loud is higher than n.

The unavailability of the generic reading expressed by (18a) would be predicted under an account of bare plural generics that involves maximality on the noun phrase. This is the case of those accounts that view bare plurals as consistently denoting kinds, such as Carlson (1977), Chierchia et al. (1995), and Dayal (2004), where kinds are obtained via a silent maximality operator '$\cap$' that, given a property, returns the maximal plural individual that has that property (at any given world). While these accounts would then straightforwardly predict (18a) to be unavailable, other competing accounts of bare plural generics, such as Diesing (1992), would require additional assumptions to rule (18a) out. Examples like (18) would then be a further corroboration of both the maximality effects observed in this paper and the 'kind-based' theories of bare plural generics. To conclude, while many areas connected to the examples presented here remain open to further understanding, I hope that the inner algebraic properties of *numerous*-like predicates and sum formation emerged from the proposed analysis, and their interaction with redundancy, can shed some light on new and existing issues, bridging broader literatures such as the one on relative clauses and - if we could ultimately establish the L-triviality component of the phenomenon presented here - the one on the logicality of language.

Appendix A

Amiraz (2021) and the *anti-monotonicity* constraint

While I am certainly indebted to Kuhn (2020) for the stress on mereological properties, Amiraz (2021) also bears interesting connections to the analysis presented here. Amiraz (2021) proposes a solution aimed at solving both the overgeneration and undergeneration problems of Kuhn (2020), and he does so pairing proportional quantifiers (19a) with previously undiscussed non-proportional partitive constructions (19b):

(19) a. All the students are tall/#numerous.
b. Forty of the students are tall/#numerous.

These two cases are grouped under the label of 'partitive quantifiers', where 'a partitive quantifier denotes a (possibly trivial) partition or set of partitions of A into objects that are in B and objects that are not in B' (Amiraz, 2021). The gist of Amiraz's analysis is that a predicate can appear in the predicative position of partitive constructions only if it can give rise to a well-formed partition and *numerous*-like predicates fail in generating one.[14] An interesting connection between Amiraz's (2021) account and the restrictive relative clauses puzzle introduced in §1 can be drawn, assuming that restrictive relative clauses - and as we've seen restrictive modifications in general - are (similar to) partitive quantifiers. The parallelism can be quite intuitive. Consider a sentence like (20):

(20) I greeted the people that went to Ireland.

The restrictive relative clause in (20) introduces two groups of people - those that went to Ireland and those that did not go to Ireland - at least in the same general sense in which partitive quantifiers introduce two groups of

[14] As said in section §2, *numerous*-like predicates are 'blind to differences', in the sense that they hold for a group as such, most of them only by virtue of the cardinality/proportion of the group. This implies the anti-restrictiveness of *numerous*-like predicates. Given a certain set, several different subsets of that set can be formed and be described as *numerous* because only the number of the elements in these subsets matters to satisfy the property. This means that given a set S, in set terms, or an individual x, in mereological terms, we can find at least two covers C_1, C_2 for which $C_1/\sim_P$ and $C_2/\sim_P$ are defined for $P =$ *be numerous* but for which $\bigoplus C_{1_{|P}} \neq \bigoplus C_{2_{|P}}$ (where $C_1/\sim_P$ is the partition of C_1 into the subset of elements in P, namely $(C_{1_{|P}})$, and the subset of elements not in P, namely $(C_{1_{|P^c}})$.). But this clearly goes against the definition of well-formed partition presented in Amiraz (2021), showing that *numerous* cannot give rise to a well formed partition, and thus cannot be felicitously used with partitive quantifiers.

people for Amiraz (2021). For this reason, under Amiraz's (2021) definition, restrictive relative clauses can be regarded as a sort of 'partitive quantifiers', since *the As that (are) B* 'denotes a (possibly trivial) partition or set of partitions of A into objects that are in B and objects that are not in B'. The answer to the unacceptability of restrictive uses of *numerous*, would be linked to its failure in giving rise to a well-formed partition. On the other hand, however, modifying *students* with *gathered* would provide a uniquely salient cover, namely the various groups of students that gathered together. This would avoid the previous issue, creating a good environment for *numerous* to define a well-formed partition. So this creates now a problem for Amiraz (2021). Consider the following:

(21) a. ??All the gathered students are numerous.
b. #Forty of the gathered students are numerous.

Even though (21a) and (21b) feature *gather* in attributive position and *numerous* in predicative position, exactly like (5), they are infelicitous. This is not expected if we just assume the puzzle to be linked to the ability of giving rise to unique partitions. At the core of Amiraz's (2021) analysis there is however a very interesting intuition. This intuition is the empirical generalization that certain predicates obey a non-monotonicity constraint, defined by Amiraz as follows:

Definition 6 (The non-monotonicity constraint) *A gradable predicate can co-occur with a partitive quantifier if and only if the dimension of the predicate is non-monotonic with respect to the dimension of the partitive quantifier.*

Amiraz (2021)

The monotonicity constraint refers to the necessary connection between the dimension of certain predicates (which in the case of *numerous/few in number/exactly two* is cardinality) and the dimension of sum formation operator (also cardinality, in the cases explored here). This generalization is indeed connected to the algebraic properties - permutation invariance and anti-restrictiveness - that I presented in the paper. However, the non-monotonicity constraint, as well as the partition requirement, can *only* be seen as mere generalizations, and the real principled explanation has to be searched - I believe - in the interaction between the essential algebraic properties of pluralities.

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